



COMMON CORE STATE STANDARDS FOR

# Mathematics

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Appendix A:

Designing High School  
Mathematics Courses  
Based on the Common  
Core State Standards

## Overview

The Common Core State Standards (CCSS) for Mathematics are organized by grade level in Grades K–8. At the high school level, the standards are organized by conceptual category (number and quantity, algebra, functions, geometry, modeling and probability and statistics), showing the body of knowledge students should learn in each category to be college and career ready, and to be prepared to study more advanced mathematics. As states consider how to implement the high school standards, an important consideration is how the high school CCSS might be organized into courses that provide a strong foundation for post-secondary success. To address this need, Achieve (in partnership with the Common Core writing team) has convened a group of experts, including state mathematics experts, teachers, mathematics faculty from two and four year institutions, mathematics teacher educators, and workforce representatives to develop Model Course Pathways in Mathematics based on the Common Core State Standards.

In considering this document, there are four things important to note:

1. The pathways and courses are models, not mandates. They illustrate possible approaches to organizing the content of the CCSS into coherent and rigorous courses that lead to college and career readiness. States and districts are not expected to adopt these courses as is; rather, they are encouraged to use these pathways and courses as a starting point for developing their own.
2. All college and career ready standards (those without a +) are found in each pathway. A few (+) standards are included to increase coherence but are not necessarily expected to be addressed on high stakes assessments.
3. The course descriptions delineate the mathematics standards to be covered in a course; they are not prescriptions for curriculum or pedagogy. Additional work will be needed to create coherent instructional programs that help students achieve these standards.
4. Units within each course are intended to suggest a possible grouping of the standards into coherent blocks; in this way, units may also be considered “critical areas” or “big ideas”, and these terms are used interchangeably throughout the document. The ordering of the clusters within a unit follows the order of the standards document in most cases, not the order in which they might be taught. Attention to ordering content within a unit will be needed as instructional programs are developed.
5. While courses are given names for organizational purposes, states and districts are encouraged to carefully consider the content in each course and use names that they feel are most appropriate. Similarly, unit titles may be adjusted by states and districts.

While the focus of this document is on organizing the Standards for Mathematical Content into model pathways to college and career readiness, the content standards must also be connected to the Standards for Mathematical Practice to ensure that the skills needed for later success are developed. In particular, Modeling (defined by a \* in the CCSS) is defined as both a *conceptual category* for high school mathematics and a *mathematical practice* and is an important avenue for motivating students to study mathematics, for building their understanding of mathematics, and for preparing them for future success. Development of the pathways into instructional programs will require careful attention to modeling and the mathematical practices. Assessments based on these pathways should reflect both the content and mathematical practices standards.

## The Pathways

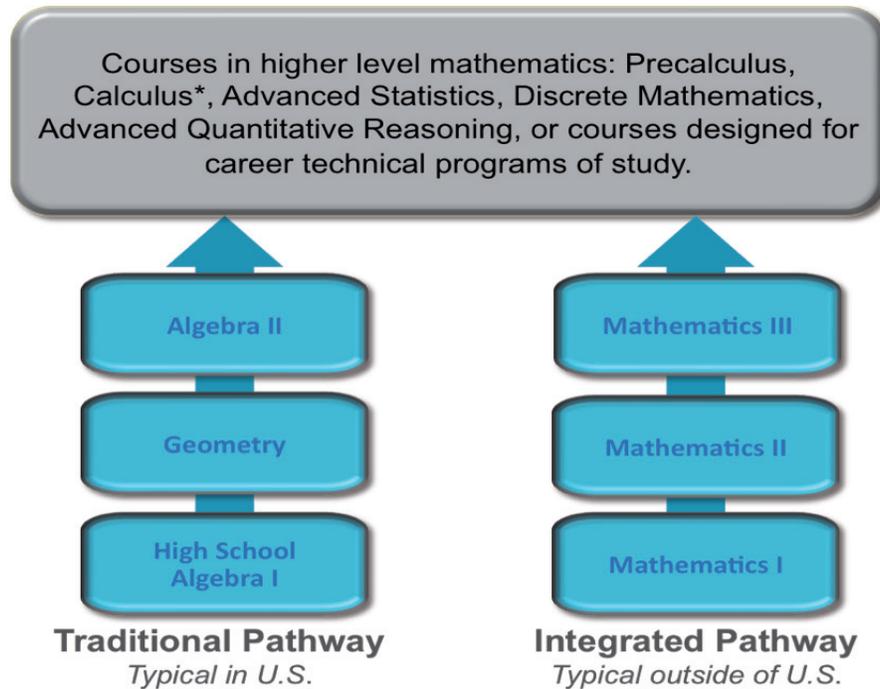
Four model course pathways are included:

1. An approach typically seen in the U.S. (Traditional) that consists of two algebra courses and a geometry course, with some data, probability and statistics included in each course;
2. An approach typically seen internationally (Integrated) that consists of a sequence of three courses, each of which includes number, algebra, geometry, probability and statistics;
3. A “compacted” version of the Traditional pathway where no content is omitted, in which students would complete the content of 7<sup>th</sup> grade, 8<sup>th</sup> grade, and the High School Algebra I course in grades 7 (Compacted 7<sup>th</sup> Grade) and 8 (8<sup>th</sup> Grade Algebra I), which will enable them to reach Calculus or other college level courses by their senior year. While the K-7 CCSS effectively prepare students for algebra in 8<sup>th</sup> grade, some standards from 8<sup>th</sup> grade have been placed in the Accelerated 7<sup>th</sup> Grade course to make the 8<sup>th</sup> Grade Algebra I course more manageable;
4. A “compacted” version of the Integrated pathway where no content is omitted, in which students would complete the content of 7<sup>th</sup> grade, 8<sup>th</sup> grade, and the Mathematics I course in grades 7 (Compacted 7<sup>th</sup> Grade) and 8 (8<sup>th</sup> Grade Mathematics I), which will enable them to reach Calculus or other college level courses by their senior year. While the K-7 CCSS effectively prepare students for algebra in 8<sup>th</sup> grade, some standards from 8<sup>th</sup> grade have been placed in the Accelerated 7<sup>th</sup> Grade course to make the 8<sup>th</sup> Grade Mathematics I course more manageable;
5. Ultimately, all of these pathways are intended to significantly increase the coherence of high school mathematics.

The non-compacted, or regular, pathways assume mathematics in each year of high school and lead directly to preparedness for college and career readiness. In addition to the three years of study described in the Traditional and Integrated pathways, students should continue to take mathematics courses throughout their high school career to keep their mathematical understanding and skills fresh for use in training or course work after high school. A variety of courses should be available to students reflecting a range of possible interests; possible options are listed in the following chart. Based on a variety of inputs and factors, some students may decide at an early age that they want to take Calculus or other college level courses in high school. These students would need to begin the study of high school content in the middle school, which would lead to Precalculus or Advanced Statistics as a junior and Calculus, Advanced Statistics or other college level options as a senior.

Strategic use of technology is expected in all work. This may include employing technological tools to assist students in forming and testing conjectures, creating graphs and data displays and determining and assessing lines of fit for data. Geometric constructions may also be performed using geometric software as well as classical tools and technology may aid three-dimensional visualization. Testing with and without technological tools is recommended.

As has often occurred in schools and districts across the states, greater resources have been allocated to accelerated pathways, such as more experienced teachers and newer materials. The Achieve Pathways Group members strongly believe that each pathway should get the same attention to quality and resources including class sizes, teacher assignments, professional development, and materials. Indeed, these and other pathways should be avenues for students to pursue interests and aspirations. The following flow chart shows how the courses in the two regular pathways are sequenced (the \* in the chart on the following page means that Calculus follows Precalculus and is a fifth course, in most cases). More information about the compacted pathways can be found later in this appendix.



Some teachers and schools are effectively getting students to be college and career ready. We can look to these teachers and schools to see what kinds of courses are getting results, and to compare pathways courses to the mathematics taught in effective classrooms.

A study done by ACT and The Education Trust gives evidence to support these pathways. The study looked at high-poverty schools where a high percentage of students were reaching and exceeding ACT's college-readiness benchmarks. From these schools, the most effective teachers described their courses and opened up their classrooms for observation. The commonality of mathematics topics in their courses gives a picture of what it takes to get students to succeed, and also provides a grounding for the pathways. (There were other commonalities. For more detailed information about this study, search for the report *On Course for Success* at [www.act.org](http://www.act.org).)<sup>1</sup>

## Implementation Considerations:

As states, districts and schools take on the work of implementing the Common Core State Standards, the Model Course Pathways in Mathematics can be a useful foundation for discussing how best to organize the high school standards into courses. The Pathways have been designed to be modular in nature, where the modules or critical areas (units) are identical in nearly every manner between the two pathways, but are arranged in different orders to accommodate different organizational offerings. Assessment developers may consider the creation of assessment modules in a similar fashion. Curriculum designers may create alternative model pathways with altogether different organizations of the standards. Some of this work is already underway. In short, this document is intended to contribute to the conversations around assessment and curriculum design, rather than end them. Effectively implementing these standards will require a long-term commitment to understanding what best supports student learning and attainment of college and career readiness skills by the end of high school, as well as regular revision of pathways as student learning data becomes available.

## Supporting Students

One of the hallmarks of the Common Core State Standards for Mathematics is the specification of content that all students must study in order to be college and career ready. This “college and career ready line” is a minimum for all students. However, this does not mean that all students should progress uniformly to that goal. Some students progress

<sup>1</sup>The study provides evidence that the pathways' High School Algebra I, Geometry, Algebra II sequence is a reasonable and rigorous option for preparing students for college and career. Topics aligned almost completely between the CCSS topics and topics taught in the study classrooms. The starting point for the pathways' High School Algebra I course is slightly beyond the starting point for the study Algebra I courses due to the existence of many typical Algebra I topics in the 8<sup>th</sup> grade CCSS, therefore some of the study Algebra II topics are a part of the pathways' High School Algebra I course, specifically, using the quadratic formula; a bit more with exponential functions including comparing and contrasting linear and exponential growth; and the inclusion of the spread of data sets. The pathways' Geometry course is very similar to what was done in the study Geometry courses, with the addition of the laws of sines and cosines and the work with conditional probability, plus applications involving completing the square because that topic was part of the pathways' High School Algebra I course. The pathways' Algebra II course then matches well with what was done in the study Algebra II courses and continues a bit into what was done in the study Precalculus classrooms, including inverse functions, the behavior of logarithmic and trigonometric functions, and in statistics with the normal distribution, margin of error, and the differences among sample surveys, experiments, and observational studies. All in all, the topics and the order of topics is very comparable between the pathways' High School Algebra I, Geometry, Algebra II sequence and the sequence found in the study courses.

more slowly than others. These students will require additional support, and the following strategies, consistent with Response to Intervention practices, may be helpful:

- Creating a school-wide community of support for students;
- Providing students a “math support” class during the school day;
- After-school tutoring;
- Extended class time (or blocking of classes) in mathematics; and
- Additional instruction during the summer.

Watered-down courses which leave students uninspired to learn, unable to catch up to their peers and unready for success in postsecondary courses or for entry into many skilled professions upon graduation from high school are neither necessary nor desirable. The results of not providing students the necessary supports they need to succeed in high school are well-documented. Too often, after graduation, such students attempt to continue their education at 2- or 4-year postsecondary institutions only to find they must take remedial courses, spending time and money mastering high school level skills that they should have already acquired. This, in turn, has been documented to indicate a greater chance of these students not meeting their postsecondary goals, whether a certificate program, two- or four-year degree. As a result, in the workplace, many career pathways and advancement may be denied to them. To ensure students graduate fully prepared, those who enter high school underprepared for high school mathematics courses must receive the support they need to get back on course and graduate ready for life after high school.

Furthermore, research shows that allowing low-achieving students to take low-level courses is not a recipe for academic success (Kifer, 1993). The research strongly suggests that the goal for districts should not be to stretch the high school mathematics standards over all four years. Rather, the goal should be to provide support so that all students can reach the college and career ready line by the end of the eleventh grade, ending their high school career with one of several high-quality mathematical courses that allows students the opportunity to deepen their understanding of the college- and career-ready standards.

With the Common Core State Standards Initiative comes an unprecedented ability for schools, districts, and states to collaborate. While this is certainly the case with respect to assessments and professional development programs, it is also true for strategies to support struggling and accelerated students. The Model Course Pathways in Mathematics are intended to launch the conversation, and give encouragement to all educators to collaborate for the benefit of our states' children.